

[Text books on Mathematics (I-VIII) published by School Education Expert Committee, WB, may always be consulted for numerical problems, method of teaching and TLM.]

Special D. El. Ed. Mathematics

Special Training (for six months)
of Inservice Elementary Teachers
having B. Ed. Training
(ODL Mode)

Pedagogy of Elementary School
Curriculum-III
Mathematics

West Bengal Board of Primary Education
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West Bengal Board of Primary Education

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Forewords

It gives me immense pleasure in presenting the materials of **Pedagogy of Elementary School Curriculum-III Mathematics for Six Month Special Training Programme in Elementary Education** for the primary school teachers in West Bengal, having B. Ed. / B. Ed. (Special Education)/ D. Ed. (Special Education). The materials being presented have been developed on the basis of the guidelines and syllabus of the NCTE.

Care has been taken to make the presentation flawless and in perfect conformity with the guidelines of the NCTE.

Lesson-units and activities given here are not exhaustive. Trainee-teachers are at liberty to plan & develop their own knowledge and skills through self learning under the guidance of the counselors and use of their previously acquired knowledge and skill of teaching.

This humble effort will be prized, if the materials, developed here in this Course-book, are used by the teachers in the real classroom situations for the development of the four skills – Listening, Speaking, Reading and Writing of the primary school children .

March-12, 2015

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Lesson Unit-1

Elementary Mathematics – Status & Issues

Lesson Unit

- 1.0 Elementary Mathematics-Status and Issues
- 1.1 Introduction
- 1.2 Objectives
- 1.3 Role of Mathematics in our Daily Life
 - 1.3.1 Role of Arithmetic
 - 1.3.2 Role of Geometry
 - 1.3.3 Higher Education and Profession
 - 1.3.4 Modern Technology and use of Mathematics
- 1.4 Exercise
- 1.5 Problems of Mathematics Teaching at Primary Level
 - 1.5.1 Removal Mathematics Phobia
 - 1.5.2 Exercise
- 1.6 Interest and Motivation in Mathematics
 - 1.6.1 Opinions of Psychologist in the development Motivation
 - 1.6.2 Motivation: Goal directed and Continuous
 - 1.6.3 Diagnosis of Disability in Mathematics and their Removal
 - 1.6.4 Exercise
- 1.7 Summary
- 1.8 Check your progress
- 1.9 More exercise

1.1 Introduction

RTE Act 2009 indicating Right to Free and Compulsory Education for all students up to 14 years has been introduced in 2009. It has been stated there that all students have right to receive their education from the duly qualified teachers. This statement is equally applicable in the teaching and learning of Mathematics. According to NCF 2005, the education should be given to the students to enable them to maintain their daily life smoothly. Every student will learn mathematics without fear to acquire necessary competence for his/her daily life. The mathematics teachers should motivate the students in this context.

1.2 Objectives

At the end of this lesson you will come to know

- Importance of mathematics in daily life
- Importance of mathematics to the ordinary as also talented students
- The problems of teaching learning of mathematics from the view points of learners, teachers, schools and curriculum.
- Different ways of inculcating interest among the elementary students.

1.3 Role of mathematics in daily life

Mathematics plays a significant role in the life of a learner just after his vernacular. Mathematics has distinguished and indispensable role in the rural and urban life, life of rich or poor. Role of mathematics is very much significant in the domestic life as well as in professional life.

1.3.1 Role of Arithmetic

In the professional sphere potter, labour, tailor, masons, carpenter use mathematics, use numbers. On the other hand distinguished professionals like engineers, businessmen, industrialists are dependent on mathematics for their works. Mathematics is categorically important to ascertain the wages of labours, study time tables of bus and rail, electric and telephone bills and to take measures accordingly. For the measurements and keeping accounts mathematics is necessary. Mathematics is inevitable in bank, post office, insurance services etc.

1.3.2 Role of Geometry

Geometry is essential in manufacturing, planning a design, a building, making furniture, planning roads.

1.3.3 Higher Education & Profession

Not only for counting and auditing, mathematics has to play a greater role in higher education and professional life of a student. Science education largely depends on mathematics. Use of mathematics is growing in the area of measurement of time, weight, volume, numerical analysis etc.

1.3.4 Modern Technology and use of mathematics

Many of us think that mobile and calculators help us to calculate and therefore mathematics is not so important for learning. It should be remembered in this context that it is mathematics that forms the base of those sophisticated instruments. So mathematics cannot be neglected.

1.4 Exercises

1. Name five professions where mathematic is indispensable.
2. Why does a tailor require mathematics?
3. Why does a farmer require mathematics?
4. Why does an engineer require mathematics?

1.5 Problems of teaching mathematics

The problems may be divided into 5 categories

(1) Problems related to learners

- i) Heterogeneous composition of the class
- ii) Presence of large no. of 1st generation learners
- iii) Mathematics phobia of the learners
- iv) Lack of initiative among the learners
- v) Long absence from schools
- vi) Disinterest in mathematical calculation
- vii) Wide differences in the techniques of learning at home and learning in schools

(2) Problems related to teachers

- i) Lack of teacher training
- ii) Lack of attitude towards mathematics
- iii) Teachers teaching mathematics sometimes without necessary expertise

(3) Problems related to schools

- i) Shortage of teachers
- ii) Lack of equipments
- iii) Shortage of teachers
- iv) More than one class in the same room
- v) Defective evaluation encouraging rote learning
- vi) Formative evaluations not always systematic

(4) Process oriented difficulty

- i) Bengali pronunciation of counting numbers makes difficulty
(mark inconsistency in pronouncing 22,42,52 in Bengali)
- ii) Learners fail to distinguish prime & odd numbers; even & composite numbers
- iii) Learners fail to find the significance of the place value of 0.
- iv) It is extremely difficult make them understand the utility of 0 when used as in 0.1 & 12.00
- v) 0.12 – why do we pronounce it as point one two instead of point twelve after the decimal.
- vi) Difference cannot be well understood between 4×6 and 6×4 and like

(5) Difficulty in using mathematics text books

- i) Learners fail to discern large assembly of charts ,diagrams, sketches in the books
- ii) Verbal ability of the students is limited. So without teacher's help they cannot understand some places.
- iii) Letter font used by the students is larger than the space provided on the workbook/textbook
- iv) In 'Search of a number after loss' (class III, Math, page-195), ordinary students cannot come to a height of creativity to solve them.

1.5.1 Removal of Mathematics Phobia:

To remove mathematics phobia following actions may be taken

- i) To encourage the guardians to send their ward/learners to schools regularly
- ii) To impart experience based lessons.
- iii) To use computers, projectors, TLM, activity based learning ,development of rapport between teacher and the learners. Formative evaluation should be more emphasized.

1.5.2 Exercise

Write the answer of the following each within 30 words :

- Mention four causes for mathematics phobia among students
- What measures are to be taken to remove them?
- Find the difficulties of the schools causing inconvenience in teaching and learning mathematics.

1.6 Motivating the children to learn strategies of training in mathematics

Important points are noted down

- To make the students attentive. [To do that the language of mathematics should be simple, past experience of the students should be exploited. Learner's environment should be used in teaching mathematics].
- To develop the self confidence of the students
- To develop the analytical thinking of the learners e.g. What is meant by $13 \times 4 = ?$ $3 \times 4 = ?$, $4 \times 3 = ?$, $0 \times 3 = ?$, $3 \times 0 = ?$
- To show the systematic steps of reasoning
- To encourage the students to freely express their difficulties in learning mathematics
- To revise lessons
- To encourage solving a particular problem in a number of ways
- To encourage discussion
- To ensure that none of the students are baffled due to repeated failure

1.6.1 Principles of Psychologists on Motivation

Cognitive psychologists like Jean Piaget and J Bruner suggested the use of concrete objects and activities in learning. Piaget has mentioned the stages of sensorimotor & concrete operation and Bruner mentioned enactive and Iconic stages in primary learning. The steps can also be used in teaching learning of mathematics also. Different educational technologies may also be used in this respect for concretizing different experiences.

1.6.2 Motivation-Goal and continuity.

To maintain the motivation of the students it should be goal directed and sustained. Teacher should see that the students do not stay away from practice of mathematics.

1.6.3 Diagnosis of Learning Disability in Mathematics and to remove them.

For diagnosing the disability of the learners more importance is to be given on formative evaluation in classroom situation. Apart from this exchange of views, board works, educational technology etc may be used to remove the weakness of the students in mathematics. To remove the disability ET(Educational Technology) may also be used.

1.6.4 Exercise

Write True/False against each of the statements:

- To develop the motivation of the students in mathematics they may be taught mathematics occasionally.
- There is no scope of discussion in mathematics
- Analytical thinking forms the base of mathematics education.
- Self confidence is the essential component of mathematics learning

1.7 Know your progress

- Write three utilities of mathematics in the home environment.
- How does an ordinary teacher use mathematics in his personal life?(Write in thirty words)
- What are the causes of mathematics phobia?
- How would you remove mathematics phobia of the students? (in 250 words)
- Write the characteristics of a teacher who can motivate his/her students towards mathematics(30 words)
- Write the characteristics of a student motivated in mathematics(30 words)
- Student having motivation in mathematics practises intensively it once in a week.

1.8 Summary

You have come to learn from the above discussion:

- Use of mathematics in everyday life
- Problems of teaching mathematics at primary level from the stand point of mathematics Phobia and failure in mathematics and absence of requisite training of the teachers.
- Motivating students in mathematics

1.9 More Exercises

- How do the distinguished professionals use mathematics in daily life?
- How do the lay men use mathematics in daily life?
- Name five professions involving measurements.
- Mention five steps for motivating students.

Lesson Unit-2

Pedagogy of teaching Mathematics at Elementary stage

Lesson Unit

- 2.0 Method of Teaching mathematics
- 2.1 Introduction
- 2.2 Objectives
- 2.3 Enjoyable Mathematics
 - 2.3.1 Psychology based Teaching learning of Mathematics
 - 2.3.2 Use of Psychology and Reason
 - 2.3.3 Example
 - 2.3.4 TLM for learning joyful Mathematics
 - 2.3.5 Activities for learning joyful Mathematics
 - 2.3.6 Exercises
- 2.4 Development of Competency in Mathematics
 - 2.4.1 Exercises
- 2.5 Method dependent on direct Experiences
 - 2.5.1 Types of Experiences of a child
 - 2.5.2 Contrived Experiences
 - 2.5.3 Importance of Experience based Methods
- 2.6. Mathematics learning as a part of daily Experience
 - 2.6.1 Different ways of acquiring Direct Experience
 - 2.6.2 Exercises
- 2.7 Problem Solving Method
 - 2.7.1 Steps of Problem Solving
 - 2.7.2 Merits of Problem Solving Method
 - 2.7.3 Demerits of Problem Solving Method
 - 2.7.4 Development of Problem and Techniques of Solution
 - 2.7.5 Exercises
- 2.8 Use of direct Experience and Educational Technology in Teaching
 - 2.8.1 Use of TLM & Educational Technology
 - 2.8.2 Summary
- 2.9 Exercises
- 2.10 Check your Progress
- 2.11 More Exercises

2.1 Introduction:

There are multifarious methods of teaching learning of Mathematics. But only that method should be used, which is enjoyable to the students, which can develop necessary competencies among the students & help solving problems. There are many narrow methods which are better in some occasions but unsuitable in other cases. But constructivist approach is an embodiment of different methods.

2.2 Objectives:

At the end of the lesson you will come to know :

- How the students learn mathematics joyfully
 - How the competencies may be acquired through learning of primary mathematics
 - How mathematics can be made an organic part of daily activities, thinking and experience.
 - How problems of life and environment can be solved by mathematics.
 - Use of concrete experience and educational technology in learning mathematics.
-

2.3 Learning to enjoy mathematics

2.3.1 Psychology based teaching and learning of Mathematics:

If mathematics learning is to be enjoyable it is to be executed according to interest, ability and development of the child. The curriculum of school mathematics should be child centric. The load on the memory of the children should be minimized. In the learning of mathematics there should be ample scope of concrete materials and activities. Learner should be given freedom in learning. Teaching should be organized from concrete to abstract. Learning should not be confined to school only, rather it will be extended to play ground, association with friends, outreach activities and leisure time activities.

2.3.2 Use of psychology and reasoning:

Mathematics is a systematic subject. For development of mathematics and learning the subject use of reasoning is very much essential. In spite of these established facts, it is admitted that if teaching learning of mathematics are to be effective it should be directed towards the need and interest of the children. Sometimes there is confusion on as to what should be taught at first: addition or subtraction. Our traditional view is to teach addition first and then subtraction. If learner has acquired the concept of 'greater than (>) and less than (<)' subtraction may precede addition easily.

2.3.3 Illustration of learning mathematics in enjoyable way:

Suppose, our main purpose of teaching is to enable the children to subtract 6 from 9. Without using the language subtraction teacher may state the problem in the following way: How many birds should join the team of 6 birds to make them 9. To solve the problem students can add 3 to 6 or they may subtract 6 from 9.

2.3.4 Materials for learning enjoyable mathematics :

The problems of mathematics are to be selected from the daily life of the children. Problems involving toys, no. of brothers and sisters; different facts related to neighbouring schools-no. of rooms, windows, benches, tables, almirahs, calendars; different types of games. The children possess two types experiences: direct concrete experience of the environment and indirect and contrived experience that is given to students

in the form pictures, diagrams, sketches etc .To make learning enjoyable concrete problems may be given to the children . Types of questions generally asked to the students may be slightly changed. To do that we may ask the questions: How many flowers are there if we add 9 flowers to 5 (five) flowers. Flowers are very favourite to the children and hence flowers have been used in lieu of any other things.

2.3.5 Activities for learning enjoyable mathematics

To make the teaching learning of mathematics enjoyable different types of games & work experiences may be arranged for the children:

- What is measure of the angle at each corner of a book?
- Shyamal is aged 70 years and Kamal is 50 years. Who will address by dada and whom? Why?
- Which is greater between Rs 40 and Rs 50? Why?
- Which is greater between Rs 50 and Rs 400? Why?
- At which time the angle between hour and minute hands become acute angle- 3pm or 3-30 pm?
- There are two points A & B at end of a diameter of a circle. What will be the least distance in coming from A to B?
- Using fingers show 5, 7,10,15,18?
- By tape they will measure different things and mention them with proper units.

Puzzles for Mathematics:

There are some problems which are simple but seem to be meaningless at first sight:

A man was alive upto $1*3$ years. If his years of living is divisible by 3, what was the minimum life span of the person? To do this use suitable digit at the place of the star mark.

Drawing diagram and use origami:

The students would put different numbers on different tables and on different diagrams ,add or subtract them, multiply or divide them to arrive at the desired solution. By folding paper they will produce different figures and diagrams.

2.3.6 Exercises

Questions:

- The height of a man of standard size is 7m/0.7m/1.5m/15m
- What is the area of the surface of your workbook ? Answer by measuring it.

2.4 Learning the basics of Mathematics at mastery level

A number of students leave after the primary education and never proceed for next stage of education. It is therefore incumbent upon the society and the schools to develop appropriate competencies among the primary students in mathematics so that they can meet the challenge of daily life even after dropping schools. These competencies involve counting, use of number for different purposes along with some basic processes in mathematics-four rules, measurement, use of units, fractions, percentages, proportion, LCM, HCF, Profit and loss etc

2.4.1 Exercise:

Questions (Write within 25 words)

1. Mention 5 competencies in mathematics.
2. What are the basic competencies which every child has to master particularly those who discontinue after elementary education.

2.5 Direct Experience Based Method

Every day and every moment a child gathers direct experience from his own environment like home, society, school etc. In his world of experience places of concrete Number and Measurement are to be identified and this will be utilized in the learning of mathematics. Each problem of mathematics is to be selected from the past experience of the students.

It is to be kept in mind that experiences of a child at this age are haphazard, scattered and not systematic.

2.5.1 Types of Experience of A Child

Experience is of two types:- (a) Direct & (b) Mock

(a) Direct Experience

- (i) Domestic affairs including number of brothers, sisters, relatives etc.,
- (ii) Domestic animals & birds, Agricultural land,
- (iii) Playing materials,
- (iv) No. of students of the school,
- (v) Sports & games,
- (vi) Observation.

On the basis of above experience a child may have conceptions like tall/short, heavy/light and equal/unequal, conception of number, conception of addition & subtraction, multiplication/division, average, fraction, measurement and geometric figures.

2.5.2 Mock Experience

By shopping, a child recognizes coin, weight and also learns to keep accounts. Through post office projects he recognizes 'what is envelope and what is post card'. Again on the basis experience of shopping in small shops with family members the following teaching may be given to the child in the home environment:-

- (a) Acquaintance with coins and notes.
- (b) Counting of coins and notes.
- (c) Addition and subtraction.
- (d) Weights and measurement.
- (e) Determination of prices of commodities etc.

2.5.3. Importance of Experience Based Method

- (i) Utilizing the practical experience of a child, teaching from concrete ideas to abstract ideas may be easily given.

- (ii) As the lessons are based on experience of the child he becomes eager to learn subject like mathematics without fear.
- (iii) As a result, learning becomes clear, meaningful and longlasting.
- (iv) Experience in the domestic environment becomes integrated with teaching-learning in the school environment of child and therefore, he/she can apply the learning outcome in the practical life.

2.6 Teaching Mathematics as a part of daily Experience of Children

2.6.1 Different methods for acquiring direct experiences

- (1) Project method
- (2) Activity oriented teaching
- (3) Different study Materials- Concept of number, Addition, Subtraction in class one, concept of plane in class four
- (4) Constructivism
- (5) Problem solving method

2.6.2 Exercise

Questions (Answer within 25 words)

- Mention six direct experience of the children
- If a child has no requisite experience how would you help him/her acquire that experience?

2.7 Problems Solving Method

Problem solving method is a method which solves the verbal or the symbolic mathematical problem.

2.7.1 Phases of Problem Solving.

- (a) Acceptance of child's problem: - According to a child's maturity level he will accept the practical and life-centric problem being inspired by the teacher.
- (b) Analysis of the Problem: - He will analyze the problem in some parts. He will observe the problem meticulously (What is given and what is wanted) and then proceed for the solution of the problem.
- (c) Data collection and the application: - The child will find out the probable ways of solution of the problem on the basis of his experience and other acquired experience.
- (d) Inference: - If appropriate methods are applied the problem can be solved.
- (e) Justifiability of the inference:- This inference shall have to be applied to many other problems of the same nature and the result would be of same nature.

2.7.2 Advantages of Problem Solving

- (a) Importance is given on the active participation of the child.
- (b) Power of reasoing and judgement is developed.

- (c) Ability of application in the practical life increases.
- (d) Increases power of decision taking.
- (e) Learning becomes clear, lucid and long-lasting.

2.7.3 Disadvantages of Problem Solving Method

- (i) There is a probability of wastage of time.
- (ii) This method is not applicable to a child at the immature stage.
- (iii) There may be probability of disturbance in the teaching process in the class.
- (iv) Child may be derailed.

2.7.4 Examples

- (a) Problem: Is it true that numbers divisible by 3 are also divisible by 6? Analysis of the problem:
 - (i) Choose the numbers divisible by 3.
 - (ii) Choose the numbers divisible by 6.
 - (iii) Whether numbers divisible by 6 are also divisible by 3?
 - (iv) Whether numbers divisible by 3 are also divisible by 6?

2.7.5 Exercises:

1. What are advantages teaching by setting problems?
2. What are the stages of problem solving?
3. Give examples for problem solving method.
4. Set a problem involving four processes of mathematics

Collection of data

- (i) Remember the multiplication table of 3.
- (ii) Remember the multiplication table of 6.

On analysis of the data and their application it appears that the numbers divisible by 3 are 3, 6, 9, 12, 15, 18, 21 etc. On scrutiny it is seen that the numbers like 6, 12 and 18 are divisible by 6 also, but the numbers 3, 9, 15 and 21 are not divisible by 6, because number 6 has factors 2 and 3, so the numbers like 3, 9, 15, 21 are divisible by 3, not by 2.

Inference: - Numbers divisible by 3 are not always divisible by 6.

The teacher may proceed with some other example for the solution of the problem for the analysis of this problem and to arrive at inference.

2.8 Use of Concrete experience and educational Technology in teaching mathematics

At the outset of mathematics learning it is necessary to get acquaintance with digits 1-9. In the earlier periods the digits were introduced with the help of some ideas in Hindu religious scripts but they not so concrete to the students-e.g 3 stands for three eyes of god, 4 for 4four Vedas, 5 for five salvos etc.

At present the digits are presented with concrete materials independent of any culture, as shown below:

- 1- head,
- 2-eyes,
- 3- wheels of a rickshaw,
- 4- legs of a table,
- 5-fingers of a hand,
- 6- points on a dice.

For greater digits we can take the help of fingers, pencils, books, beads of necklace. Moreover we can take the help of match sticks, abacus etc. Intrinsic value and place value of a number can also be taught to give the concept of greater than and less than. The idea of odd and even numbers can be given with help of the previous experience. Idea of fraction can be given with help of dividing food, fruits, breads etc. The idea of LCM & HCF can be given on the basis of those divisions. On the basis of the experiences of the students time, duration, velocity, path described etc can be discussed.

With the help of a currency notes, rectangles, angles, cone, different numbers of rods of windows, banana trunks, bread, plate etc one can demonstrate different geometrical figures. Solid bricks/ marbles may be used to furnish geometrical ideas. Table clocks can be used to present the idea of circle, angle and radius. Branches of trees or inclination at elbows can demonstrate angles.

2.8.1 List of TLMs

For each class a box made of papers to be kept. In this box necessary materials shall be kept according to the list of different materials which will be used in teaching the subject content of mathematics.

Sl.	TLM	CLASS	To be used in teaching mathematics
(1)	Earthen small round ball addition and subtraction	I	Conception of number in teaching additions and subtraction.
(2)	Stick	I	Do
(3)	Stone & stone chips collected from different places from sizes.	I	Do
(4)	Different pictures.	I	Do
(5)	Number card.	I	Do
(6)	Cluster made up of 10 and 100 units.	I & II	Conception of 10's place and 100's place.
(7)	Abacus.	I	Conception of addition and subtraction.
(8)	Small containers and phials made of glass of different sizes.	I, II & III	Conception of measurement of quantity of liquids, conceptions of weight of different materials.

SI.	TLM	CLASS	To be used in teaching mathematics
(9)	Tape made of cloth and meter scale.	I & V	Conception of meter / centimeter.
(10)	Beam of balance.	I & II	Conception of weight.
(11)	Earthen weights.	I & II	Unit of weight.
(12)	Broken brick.	I & II	Conception of unequal surfaces.
(13)	Small round shaped containers or funnels.	IV	Conception of curve and spheres.
(14)	Triangle, quadrilateral, circle and rectangle made of cardboards.	V	Conception of different geometrical shapes.
(15)	Coloring the cardboards	III & IV	Conception of fractions.
(16)	Number chart and multiplication table chart made of art-paper.	I, II & III	Conception of addition and subtraction.
(17)	Playing with number made of art-paper.	I & II	Do
(18)	Different cubes and balls made of soil.	IV & V	Conception of sphere, rectangle-cube and cylinder.
(19)	Base board.	IV	Conception of LCM

2.8.2 Exercise

- Some spherical bodies are used in some games. Mention them.
- Circular discs are required in some games. Name them.
- Sikha possesses Rs.40 and Amina Rs 120. If Sikha posses one parts how many parts are possessed by Amina?
- Some concepts are noted below. Suggest TLM for them:
 1. Idea about 10's & 100's place
 2. Addition & subtraction
 3. Idea of irregular surface
 4. Idea LCM & HCF
 5. Idea of fraction

2.9 Know your Progress

- Mention three methods by which you can make your lesson in mathematics enjoyable
- What competencies should be acquired by the students of your locality, who will not continue study after primary stage?
- How would you proceed to make mathematics an integral part of life process?

2.10 Summary

From this unit you have come to learn :

- How a student learns mathematics joyfully ?
- Basic competencies at mastery level
- Learning mathematics as a part of daily life
- Presentation of problems and the methods of solution
- Use of direct experience and educational technology in learning mathematics

2.11 Exercises

- Compare the competencies of a mason and a carpenter.
- Discuss the role of Mathematics in the life of a street child.
- How can you teach mathematics to a student at a market place .
- Enumerate the competencies to be possessed by students living near a river.
- Folding a paper how can you demonstrate different geometrical figures?

Learning Unit-3

(Numbers and Four Operations)

Lesson Unit

- 3.0 Number & Four process
- 3.1 Introduction
- 3.2 Objectives
- 3.3 Numbers and numerals
 - 3.3.1 Real Value of a Number
 - 3.3.2 Place Value of Numbers
 - 3.3.3 Nominal Value of Numbers
 - 3.3.4 Cardinal & Ordinal Value of Numbers
- 3.4 Four operations
 - 3.4.1 Process of Addition
 - 3.4.2 Process of subtraction
 - 3.4.3 Process of Product
 - 3.4.4 Process of division
- 3.5 Pedagogy of 4 Processes
 - 3.5.1 Process of Addition
 - 3.5.2 Process of Subtraction
 - 3.5.3 Process of Product
 - 3.5.4 Process of Division
- 3.6 Factors and Multiples
 - 3.6.1 Concept of Prime & Composite numbers with the help of factors
 - 3.6.2 Distinction between Factors and Multiples
 - 3.6.3 Mutual Prime Numbers
- 3.7 Different types of numbers
 - 3.7.1 INTEGARS AND FRACTION
 - 3.7.2 Other numbers
- 3.8 Apprimation and estimation
 - 3.8.1 ESTIMATION
- 3.9 Application of numbers and four Principles
- 3.10 Check your Progress
- 3.11 Summary
- 3.12 Exercises and some more questions

3.1 Introduction

To days' society is based upon information and technology where mathematics plays a predominant role. That is why, every country has included mathematics in school curricula. In our country also mathematics plays a dominant role in school mathematics curriculum. Mathematics is a reasoning based subject. From the very beginning mathematics should be made life centric, scientific, systematic and psychological in our schools. We know that every normal child possesses the ability of learning mathematics. Arithmetic, a part of mathematics, involves numerals. So numerals and the four processes form corner stone of mathematics.

3.2 Objectives

After pursuing this unit you will come to learn

- (1) Concept of number and place value of numbers.
 - (2) Four operations in arithmetic.
 - (3) Classification of numbers and their comparison.
 - (4) Rule of divisibility
-

3.3 Cardinal and Ordinal Aspects of Number

3.3.1 Real Values of Number

Numbers particularly from 0 to 9 can be explained in two ways .

Let numbers from 0 to 9 be written on paper blocks. If the students are asked to arrange the numbers systematically, the students who have understood the numbers will arrange the numbers hierarchically from 0 to 9, though 0 will indicate nothing in this perspective. So $0 < 1 < 2 < 3 < 4 \dots < 8 < 9$. Here intrinsic value of 1 is always greater than 0 and 2 greater than 1. Here size of 2 or the quantity expressed by 2 is always greater than 1. This value of number is the cardinal/intrinsic aspect of number.

From this point of view if we compare 40 and 50, 50 is greater because $5 > 4$, $5 \times 10 > 4 \times 10$. With similar reason $500 > 400$.

3.3.2 Place Value of Digits

But $400 > 50$ because 4 is in the hundred's place, so $400 = 4 \times 100$ but 5 is in the ten's place, so $50 = 5 \times 10$. So place value changes the strength of a digit. 4 being placed in hundred's place is greater than 5 in the ten's place.

We know $1 < 3 < 6$. But in 136, 1 represents 100(1 in the hundred's place), 3 represents 30(3 in ten's place) .Hence $6 < 30 < 100$

Third Example:

H	T	U	tenth	hundredth
1	3	6	.	5 7

The 5 in the tenth place means one tenth of 5 i.e. $5/10$ and 7 indicates one hundredth of 7 i.e. $7/100$

PLACE VALUE OF 0

If we write 0 it indicates nothing. If we write 05, 0 has no significance or value here. But when we write 50 it does no longer indicates 'nothing'. 0 placed on the right of 5 five goes to the unit's place and 5 goes to the ten's place. So value of 5 is elevated to a number ten times of 5 i.e. 50.

3.3.3 Identifying number or nominal number

House number, roll number and telephone number are all identifying number. These numbers neither represent any size nor any sequence/rank. They are simply the identifying numbers. These numbers are called nominal numbers.

3.3.4 Cardinal and ordinal aspects of number:

In cardinal aspect of numbers 1,2,3 expresses sizes/quantities. If we take numbers to indicate length, $1\text{cm} < 2\text{cm} < 3\text{cm}$ etc. Sometimes the number indicates an individual in increasing or decreasing sequence: 8th man is to be considered only after the 7th man. 8th sec is a second to come after 7th sec. The number taken to indicate this priority or sequence is called ordinal number. Sometimes order of the number indicates the quality or priority. The quality of the second prize winner is greater than that of the third one. In roman numerals the positions or orders are written as I, II,III,IV,V....(1st,2nd,3rd..)

Exercises:

- Insert '<' or '>' which suitable between 7 & 5.
- As place value which is greater between 7&5 in 57?
- While representing place value which is greater between 7&5 in 0. 57?
- Find the differences among 8 boys, 8th boy, Roll no.8

3.4 Four Operations(Nature)

Addition, Subtraction, Multiplication & Division

3.4.1 Process of addition

In addition the resulting number increases and is greater than the initial numbers (no number should be negative for this discussion). Both five and seven are positive numbers. Five friends are sitting on a moving bus. At the next stage five more friends joined them. What is the number of friends then? This kind of process of addition can be taught by direct or contrived experience. The '+' sign indicates growth. Addition may take place in different ways: e.g. addition of two or more lengths, volume, weights. It definitely indicates that addition is concerned with things of similar characteristics. 5 pens cannot be added with 4 cows. If it is seen that $5+9=14$, it definitely means that 5 & 4 have similar characteristics. Jean Piaget used the following examples for conceptualizing addition process. Two glasses of identical shape contain water 5 unit each. Then the water is transferred to a taller glass of same shape. Where water looks more? Why? The water of two glasses has been added in the third glass. In one container there are four pencils and in the other five. The pencils are transferred in a third container. What is the no. of pencils in the third container? What is the relation of the no. of pencils of the third and first container?; third and second container? No. of pencils in the third container is greater than that in each of 1st & 2nd containers.

Exercise:

1. A student studies 2 hours in morning and 3 hours at night. How much time does she spend on study? How do you get it?
2. Water contained in two glasses are poured in a bowl. Where is the quantity of water larger?
3. What is the relation between the quantity of water in bowl and any one of the two glasses?

3.4.2 Process of subtraction

Subtraction is a gradually decreasing phenomenon where something is removed from a collection of things. Example: Mithu takes away 6 sweet meat from a collection 15 in a plate? What is the no. of remaining sweet meat in the plate? Here the no. of remaining sweetmeat is less than the initial intact no. If Mithu theoretically wants to estimate the no. of sweetmeat in the plate what would she do? The process Mithu wants to utilize is the process of **subtraction**. The phenomenon of subtraction occurs in our environment in various ways. After festival, no. of relatives decreases in our house. When we distribute toffes among the friends the no. decreases. When we consume drinking water from water bottle it decreases. These are also the examples of subtraction. Oppositely when festival starts relatives comes or when we fill up bottle with water, it becomes a process of addition. Addition, thus, becomes a process opposite to subtraction.

That addition is opposite to subtraction may be given by the following example. Mantu has two books .Odud gives him three books. What is the no. of books in the possession of Mantu? (It is the example of addition).If Mantu returns the books to Odud after use. What is the no. of books in the possession of Mantu? Definitely two (Thus it is an example of subtraction).Thus subtraction process is opposite to addition. Symbolically it may be represented as $A+B=C$ (**A** added with **B** gives **C**). Hence $A= C - B$ or

$B=C-A$ (when **B** or **A** is withdrawn, **A** or **B** returns respectively)

Exercises:

- (1) Give two examples of subtraction from our daily life.
- (2) Give examples to show that addition and subtraction are the opposite processes.

3.4.3 MULTIPLICATION

Repeated sum of a number is called multiplication. We want to add a number (3) four times. Then we should write $3+3+3+3=12$. It may be written in a different way i.e. $3 \times 4=12$. Here 'x' sign is concise version of + sign. This means 3 is added 4 times or 4 times 3. Again $4+4+4=12$. It can be written as $4 \times 3=12$. This means three times 4. So $3 \times 4=4 \times 3$(1) We can state this equality as 4 times 3 is equal to 3 times 4.

Let us explore the process for multiplication with 0. 4×0 is a meaningless process. It may at best mean that 4 has not been considered at all. But when we write 0×4 , it has a meaning. Here 0 is added 4 times to give result 0. Following the eqn. (1) we may write $4 \times 0=0 \times 4$.

Exercises:

- (1) What is meant by 5×4 ?
- (2) What is meant 4×5 ?
- (3) What is meant by 0×6 ?

3.4.4 Division

In the process of division a number has to be split into a no. of equal parts. If we like to divide 12 into three equal parts, we write it as $12 \div 3$, here there is no remainder. When there is remainder the following process has to be adopted. We like divide 67 with the help of 6 so that each part of the division is an integer, in that case a remainder may be left.

$$\begin{array}{r}
 1 \quad 1 \\
 6 \overline{) 67} \\
 \underline{6} \\
 7 \\
 \underline{6} \\
 1
 \end{array}$$

Exercises:

1. What is the relation among dividend, divisor, quotient, remainder?
2. In ordinary division process which is greater-Divisor & remainder ?

3.5 Pedagogy of four operations

3.5.1 Rule of Addition

For addition the learner has to acquire the basic skills of addition

1	1	1	1	1	1	1	1	1	1
1	2	3	4	5	6	7	8	9	9
2	3	4	5	6	7	8	9	10	10
2	2	2	2	2	2	2	2	2	2
2	3	4	5	6	7	8	9	9	9
4	5	6	7	8	9	10	10	11	11

3	3	3	3	3	3	3	3
3	4	5	6	7	8	9	9

.....

6	7	8	9	10	11	12
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i.e. $1+1=2$, $1+2=3$, $1+9=10$ In these skills students learn to add different basic numbers. Here the 1st no. is smaller and the 2nd is larger. If the 1st one is larger and the 2nd one is smaller, a student has to learn

36 skills more. For adding numbers with 0 19 more skills are to be mastered like $0+0, 0+1=1, 0+2=2$ —
—In this way 100 skills are to be mastered by the children for learning addition.

Steps of learning addition

1st Step: Counting no. of beads **or** counting no. of flowers drawn in a page students may start addition.

2nd Step: Sum of one digit no. in a column

$$\begin{array}{r} 1 \\ 2 \\ 3 \\ \hline 6 \end{array}$$

Third Step: To add one digit number with two digit number so that the sum does not exceed 9 in any column.

$$\begin{array}{r} 2 \quad 3 \\ \quad 4 \\ \hline 2 \quad 7 \end{array} \qquad \begin{array}{r} 1 \quad 4 \\ \quad 5 \\ \hline 1 \quad 9 \end{array}$$

Fourth Step: Sum of two 2 digit numbers.

$$\begin{array}{r} 4 \quad 3 \\ 2 \quad 5 \\ \hline 6 \quad 8 \end{array} \qquad \begin{array}{r} 3 \quad 5 \\ 6 \quad 3 \\ \hline 9 \quad 8 \end{array}$$

Fifth Step:

Sum of two 2 digit numbers with sum of unit's place being greater than 10

$$\begin{array}{r} 4 \quad 7 \\ 3 \quad 4 \\ \hline 8 \quad 1 \end{array}$$

Sixth Step:

Sum of two 2 digit numbers with sum of ten's place equal to 10 or greater than 10

$$\begin{array}{r} 4 \quad 7 \\ 5 \quad 4 \\ \hline 1 \quad 0 \quad 1 \end{array}$$

Seventh stage: Complex addition: Sum of two digit and 3 digit numbers

$$\begin{array}{r}
 5 \quad 3 \quad 8 \\
 2 \quad 0 \quad 8 \\
 \quad 7 \quad 4 \\
 \quad 6 \quad 3 \\
 \hline
 8 \quad 7 \quad 9
 \end{array}$$

3.5.2 Subtraction

1st Step: To subtract 0 from the numbers 0 to 9

To subtract 1 from numbers 1-9

To subtract 2 from numbers 2-9

To subtract 3 from numbers 3-9

.....
 To subtract 9 from numbers 9-18

All these form in total 100 skills(for subtraction)

2nd step: Simple subtraction without borrowing

$$\begin{array}{r}
 4 \quad 8 \quad \quad 2 \quad 5 \\
 2 \quad 3 \quad \quad 1 \quad 3 \\
 \hline
 2 \quad 5 \quad \quad 1 \quad 2
 \end{array}$$

Third step (a): producing change in the process and not in the subtraction

$$83 = 80 + 3 = 70 + 13 = 7 (13)$$

$$59 = 50 + 9 = 5 \quad 9$$

$$2 \quad 4$$

(b) By changing subtrahend

$$83 = 8 \quad 13$$

$$59 = 6 \quad 9$$

$$2 \quad 4$$

3.5.3 Multiplication

Different processes are there but only two processes are important

(2) By using factors

In division by factors under this rule, the factors of the dividend are used to successively divide the dividend and ultimately the remainder is found.

$$326 \div 15$$

$$15 = 3 \times 5$$

15 has two factors

$$\begin{array}{r|l} 3 & 326 \\ \hline & 5 \overline{)108} \quad \text{--- Remainder 2} \\ & 21 \quad \text{--- Remainder 3} \end{array}$$

Quotient = 21.

Reminder = Remainder 2 + Remainder 3 \times 1st divisor

$$= 2 + 3 \times 3 = 11$$

Exercise:

- (1) How many basic skills are there for addition?
- (2) Give examples of any two of the above skills?
- (3) Give examples of any two skills of subtraction?
- (4) Why is multiplication considered as the precise version of addition?
- (5) Give an example of the process of division.

3.6 Factor and Multiple

Before giving the concept of factor and multiple, a problem may be presented to the children. Two brothers came from Delhi by Rajdhani Express. One has ticket and another has food. One boarding at Kolkata bound Rajdhani and another boarding at Howrah bound Rajdhani. First train stopped every after three stations and second train stopped four station interval. Now ask to the children where two brothers will meet.

Factors : A given number is divisible by other numbers. Then these numbers are called the factor of the given number, e.g. - the factors of 12 are 1, 2, 3, 4, 6, 12. Remember the number of factors of a number is finite.

Multiples : The numbers which are divisible by a given number are called the multiples of the given number e.g. 4, 8, 12, 16, are the multiples of 4. Remember, multiples of a number are infinite.

3.6.1 Prime and Composite number with the help of factors

The number of factors of prime number are two - 1 and that number. Number of factors of a composite number are more than two. 1 is neither prime nor composite.

Prime to each other

If there is no common prime factor of two numbers, they are called prime to each other. e.g. - (2, 3). One prime and other composite may become prime to each other. e.g. (7, 8), (5, 9) etc. Two composite numbers become prime to each other. e.g. - (8, 9), (24, 25).

Remember :-

- (a) Two consecutive numbers are always prime to each other.
- (b) Multiples of a number are divisible by its factors.

3.6.2 Difference between factor and multiple

- (a) Factors divide the number ; multiple is divisible by the number.
- (b) Factors are finite ; multiple are infinite.
- (c) Factors are lesser or equal the number ; multiples are equal or greater than the number.
- (d) 1 is the factor of every number ; 1 is not multiple of any number except 1.

3.6.3 Two prime numbers are prime to each other, but the converse is not true

There is no prime factor of a prime number other than the number itself. Again coprime number have no prime factor. So, two prime numbers are always coprime. e.g. (5, 7), (3, 7) etc.

The converse statement is not always true. e.g.-

- (a) Two composite numbers which have no common prime factor are coprime. e.g. -(8, 9), (15, 14) etc.
- (b) One composite and other prime numbers which have no common prime factor are also coprime. e.g. (7, 8), (5, 9) etc.

So, converse statement is not always true.

Exercise

- What is number of factors of 12? What are these?
- What is the number of factors of 13? What are these?
- What is the number of multiples of 12? What are these?
- What is the number of multiples of 13? What are these?

3.7 Patterns of number

3.7.1 Number may be of two types

Integer(1,2,3,4....) and fraction($1/2, 3/7$ etc)

Fraction are of two types common fraction & decimal fraction

Common fraction: When a number is split into any number of equal parts , then each split part (other than integer) are common fraction.

Proper fraction $1/3, 4/5$. Here numerator is less than denominator

Improper fraction $5/4, 7/6$. Here numerator is greater than denominator

Mixed fraction $2 \frac{1}{6}$, it is a mixture of proper fraction and integer

Decimal fraction:

1. Common decimal: When a number is split into any $10/100/1000$..equal parts, then each split part (other than integer) are decimal fraction. Decimal number starts with decimal point after unit's place: $4/10=0.4$, $4/100=.04$, $4/1000=.004$, $56/10=5.6$, $56/100=0.56$

2. Incomplete decimal= $3.73776811\dots$

3. Recurring decimal $20/3=6.6666\dots=6.\dot{6}$

3.7.2 Other types of numbers:

1. Even numbers: Numbers which are divisible by 2 are even numbers: 2, 4, 6, 8, ...

2. Odd Numbers: Which are not divisible by 2: 3, 5, 7, 9, ... (1 is not considered as odd number)

3. Prime numbers: Which are not divisible by any number other than one and the number itself. [2, 5, 7, 11, 13, 17, 19, etc]

4. Composite number: The numbers which are divisible by integral numbers in addition to one and the number itself are composite numbers. The composite numbers are 4, 12, 15, 18 etc

5. The numbers greater than 0 are **positive** numbers. Example 1, 2, 3, 4, 0.1, 2.3, 0.0001

6. The numbers less than 0 are negative numbers. Example -1, -2, -3, -4, -0.1, -2.3, -0.0001

7. Rational number: If any number or the root of a number can be expressed as the ratio of two integers (p/q), the number or the root is said to be rational. e.g $\sqrt{4}=2/1$, $9=3/1$, $4/9=2/3$, $2 \frac{1}{2}=5/2$

8. Irrational numbers $2=1.414\dots$ can not be expressed in the form p/q . So it is an irrational number.

$0.=1/3$ is a rational number.

Exercise:

1. What are proper and improper decimal? Give one example of each.
2. Give examples of common and decimal fraction.
3. Give an example of a prime number.
4. Why is 2 called a prime number?

3.8 Approximation & Estimation

Two Problems :

Approximation: Sometimes we cannot get definite quotient by division e.g. 263. The quotient = 8.66...

We have no coin of any denomination below 1 rupee. Suppose the cost of a pair of napkin is Rs 57=00

Generally we write the quotient above as 8.66. We pay Rs 28.00 for one napkin, stated above. These are not the exact values. Yet these values serve our purpose well. So these are called approximate values. $2=1.414\dots$. But for all practical purposes we write it as $2=1.414$

This is also the approximate value but not the exact one because that is unavailable. But this value is very much near the true value and serves the desired needs. So this value is an approximate value.

3.8.1 Estimation

Radhanath Sikdar, a legendary surveyor of our country, just by observing the shadow of a cliff claimed it to be the highest peak of the world. Afterwards accurate measurement was carried out and the assumption of Mr Sikdar was confirmed. This peak or cliff was later called Everest. Similarly situation may arise when actual precision measurement is not possible due to several difficulties but a measure is very much essential. In this case we make an estimation of the value of the object, be it length, energy, height, volume, time, weight/mass.. In flood emergency, we have to know length of a damaged road or a portion of tree broken. We have to find mentally and but reasonably those measures which stand very much near the correct one. We want to know the height of our class room. Sometimes measuring instruments are not available to us, time for measurement is very short and we have to be satisfied for the time being with the value of measurement on the basis of our experience and reasoning. No doubt such mental measurements may have some error.

Those who can compare mentally but effectively two sets of times, lengths, volumes, areas can more accurately state the results of measurement before the actual measurement. In estimation an unknown measure has to be compared with already measured ones or any standard measurement. It is to be mentioned that estimation is a firsthand measurement and should be followed by actual measurement as and when possible.

Questions:

1. What is approximation? What is its necessity in our life?
2. Why do you require estimation in measurement?
3. What method would you use to teach estimation?

3.9 Application of numbers & four rules in our daily life

Application of numbers:

In home ; numbers of windows, doors, books, quantities of vegetables, counting times etc

In schools: Number of students, teachers, rooms, their windows and doors, black boards, chairs

In environment: No. of birds, trees, ponds, animals, buildings etc.

In Market: No. of fruit packets, packet of salts, weight of fishes, sweet meats,

In constructing buildings: number of bricks, no. of rods, no. of flats, etc.

Application of rule of four:

Addition: In the cases of Consumption of energy daily and monthly, use of rice daily and monthly, total cost in marketing daily, monthly and annually.

Subtraction: In the cases of Income, expenditure & savings; income statement of an office, purchased & used fertilizer; transaction in Bank, Insurance & Post Offices.

Multiplication: In the cases of Payment of energy bill , monthly cost of rice, total cost for edible oil, cost of bricks for the entire building, total monthly transport cost, time and distance etc.

Division: to know the rate of expenditure, cost of travel per day per head, sharing expenditure in picnic, to find the density of matter, to find the velocity we use division.

Exercise:

1. Show how do we use four rules in marketing. Glve one example in each case.
2. How do we use multiplication and division in our kitchen?
3. How do we use division when we are on tour?

3.10 Know your Progress

1. What is cardinal aspect of number? Give an example of it.
2. With an example show subtraction is opposite to addition.
3. What is multiple and factor?
4. How many factors are possessed by a prime number?
5. What is decimal fraction? Write seven parts of hundred in a mathematical form.
6. What is positive number? Which one is greater between -3 and 0.1?
7. What is meant by approximate value of a number?
8. Show how do the four operations take place on dining table?

3.11 Summary

In this unit following concepts have been discussed:

- Number and numerals. Every number represents a value or quantity which indicates whether that number is greater or smaller than other number. This is the cardinal aspect of number. Sometimes number is used to indicate position e.g. 1st, 2nd, 3rd etc. Here position rather than value is emphasized . Sometimes numbers are used as labeling (Nominal numbers).
- Concept of addition and subtraction which are oppositely directed.
- Factors and multiples
- Different types of number-integer, fraction, decimal fraction, rational and irrational numbers etc.
- Estimation (to mention the value of some quantity by comparing with a standard one before any rigorous measurement)
- Use of numbers and four rules
- Teaching method of four rules.

3.12 More questions for exercise

- Mention two initial steps for teaching addition
- Mention five areas of life where use of number is indispensable
- Why is it more difficult to teach division process?

Lesson Unit -4 Measurement

Lesson Unit

- 4.0 Measurement
- 4.1 Introduction
- 4.2 Objectives
- 4.3 Scope of Measurement
- 4.4 General Measurement
- 4.5 Metric Units and their mutual relation
- 4.6 Solution of different Problems in daily life and their solution
- 4.6.1 Solution of Problems related to Mathematics
- 4.7 Method of Teaching Measurement
- 4.8 Summary
- 4.9 Know your progress
- 4.10 More questions
- 4.11 Exercises

4.1 Introduction

For selling and purchasing, to determine exact cause of some situation, to make a composition to make something new, to know the structure of a body, to measure velocity etc we require precise measurement. As a matter of fact we have to measure length, mass (weight), time, area, volume in most of the time.

4.2 Objectives

This unit undertakes discussion on:

- What are the advantages of measurement?
- Which of the quantities are generally measured?
- Metric system of units.
- Problems related measurement in daily life.
- Teaching of method of measurement.

4.3 Scope of measurement

In order to get precise information of an object we require measurement. In order to perform a work in a cost effective way and in reliable way measurement is a must. Earlier in our society there was no measurement and exchange system was prevalent in the society .

With the development of science and technology when economic development came into being measurement became essential even to the sundry. Measurement today is not limited to foodstuff, domestic necessities, road, transport. It is also extended to micro dimensions like electron, virus, bacteria and macro dimension like cosmic distances. Very small and very large distances are measured in laboratories and the technique of measurement is complex. In comparison to that the measurements in our daily life are simple and these are concerned with measurement of distance, time, volume, area, velocity, weight etc. These measurements include solid, liquid and gases. In order to express the result of such measurements we require value of measurement and the proper units of measurement.

4.4 Measurement in General

Among the measurable quantities six fundamental quantities are considered

1. Length
2. Mass/weight
3. Time
4. Temperature,
5. Intensity of light,
6. Strength of Electric current.

Measuring scale of 1st three quantities are available to us viz, measuring tape for length, common balance for mass & watch for time. Some more measurement are associated with are daily life : Relative Humidity, Blood pressure, blood sugar. Such measuring instruments are easily available to urban people. Electric meter help us to measure the electric energy consumed by any consumer.

4.5 Different types of metric units and relation among them

The fundamental unit for mass is kilogram, for length metre, for time second.

Units for length:

The superordinate unit of length is kilometer. The subordinate units are cm, mm. The relation among them:
1km=1000m, 1m=100cm, 1cm=10 mm

Units for mass: The super ordinate unit is metric ton & subordinate units are gram, milligram

1ton=1000kg, 1kg=1000gm, 1gm=1000mg

Time: The Super ordinate units are minutes, hours, days

1day=24 hour, 1hour =60minute, 1minute= 60 sec

Units for volume

Unit of volume is cubic metre (m³). For measuring volume of liquid generally the units used are litre, milliliter (ml)

1 cubic metre = 1,000,000 cubic cm

1 cubic centimeter = 1000 cubic millimetre

Units for area

Metric unit for area is Hectare

1 Hectare=100m x 100m=100,00 square metre

4.6 Solving problems related to measurement in our daily life

In case of any trouble arising out of measurement we can inform the appropriate authority to take necessary actions. We may take the help of physician to report anomaly in blood pressure, for any problem arising out of purchase we may approach the consumers' forum. We must have awareness regarding measurement and process of measurement.

If we come across any social problem related to measurement we may approach appropriate authorities for redresses.

4.6.1 Mathematical problems related to measurement

- 1.090litre is added to 700ml of kerosene . What is the total volume?

Solution: 700 ml = 0.700 litre So total volume = 1.090 + 0.700 = 1.790 litre

- The length of a road is 2km 400m. The road has been increased by 1.3 km. What is the total length of new road?

Solution: 1.3 km=1 km 300metre. Total length of the road = 2 km 400 m.

$$\begin{array}{r} + 1 \text{ km } 300 \text{ m.} \\ \hline 3 \text{ km } 700 \text{ m} \end{array}$$

4.7 Method of Teaching Measurement

Here we shall mention only the measurement of length, mass & time

Objectives: Students will

- be acquainted with different types of measurement
- mention different units of measurement
- be acquainted with informal measuring technique
- acquire skill of measurement

TLM: Spring balance, pen, ruler, mobile phones fitted with stop watch etc

Presentation

- Which one of the dot pens is longer?
- How much?

Teacher will discuss the method of presentation and use of ruler to measure the length. Students will participate actively in the measurement in small groups. They will discuss among them if and when necessary.

Teacher will advise the students to use appropriate as also convenient unit of length.

This technique may also be extended to the case of measurement mass and time.

Teacher will elicit responses of the students in respect of following issues:

- What more measurements do you require in your daily life?
- What measuring instruments do you require for them?
- What units do you require for each of them?

They will write in their workbooks the following in the tabular form:

Measurable quantities, corresponding measuring instruments and units.

4.8 Summary

We have already learnt

- Situations where measurements are essential
 - The measurements which we require daily
 - Different units in metric system. Relation between subordinate and super ordinate metric units e.g. the relation between metre and millimetre.
 - Method of teaching measurement and how to measure with precision.
 - Problems relating to measurement in daily life
 - Method of measurement, measuring instruments and
 - Accuracy in measurement
-

4.9 Know your progress

- Find relation among the metric units of length.
 - How many seconds contained in a solar day?
 - Mention the metric units related to volume?
 - Mention a situation where precise measurement of time is of utmost necessity?
 - In order to measure the distance between home and school mention an interesting technique to your students.
-

4.10 More questions

- Mention the different occasions where you can advise your students to measure time.
 - Write how you would teach your students addition and subtraction related to time.
-

4.11 Exercise

- Write the different scopes of measurement in our home.
- In our common life which measurements are of greatest importance to us?
- What are the measuring skills possessed by a carpenter?
- What are the measuring skills possessed by a grocer?
- Mention the different metric units used in our daily life?
- What types of measuring difficulties occur in our daily life?

Lesson Unit-5

GEOMETRY

Lesson Unit

- 5.0 Geometry
- 5.1 Introduction
- 5.2 Objectives
- 5.3 Nature of Geometric Ideas at Primary Stages
- 5.4 List of Geometrical Objects at Primary Level
- 5.5 Presentation of Lesson-Solid Bodies
- 5.6 Line
 - 5.6.1 Point
- 5.7 Concept of angle
- 5.8 Practical Works related to Classification of Objects
- 5.9 Types of Angles on the basis of Relation between them
- 5.10 Plane Area
- 5.11 Triangles and their classification
- 5.12 Practical works for development of understanding of triangles and types
- 5.13 Quadrilaterals and their classification
- 5.14 Circle and its different parts
- 5.15 Expected ***Learning Outcome***
 - 5.15.1 Line and their classification
 - 5.15.2 Angle and their classification
 - 5.15.3 Triangle and their classification
 - 5.15.4 Quadrilateral and their classification
 - 5.15.5 Circle and its different parts
- 5.16 Exercise
- 5.17 Formation of Line segment and its measurement
 - 5.17.1 Measurement of length of line segment
- 5.18 Drawing angle and its measurement
 - 5.18.1 Drawing angle
- 5.19 Drawing Triangle
 - 5.19.1 Drawing Triangle with different values of angle
- 5.20 Drawing polygon
- 5.21 Drawing diagrams with basic geometrical figures
- 5.22 Drawing models with solid figures
- 5.23 Pedagogy of spatial Relationship
- 5.24 Acquaintance with different figures
- 5.25 Acquaintance with different lines
- 5.26 Summary
- 5.27 Know your progress

5.1 Introduction

Geometry is the most important branch of mathematics. A child can develop his/her thinking ability, reasoning ability, logical ability by observing and using the mathematical property around his/her environment. The things that he/she sees are mainly solid, surface and plane area. Every person requires a minimum knowledge of geometry for measuring any type of quantity that he/she faces in daily life. Person like carpenter, massion etc are required to have minimum geometrical concept for his profession.

5.2 Objectives

After reading this chapter you will learn –

- (1) Concept of geometrical figure.
- (2) Concept of surface, types of surface.
- (3) Concept of line, line segment, angle, types of angle.
- (4) Clear concept about triangle, quadrilateral, parallelogram, circle etc.

5.3 Nature of geometrical concept at primary level

- (a) The concept should be child centric.
- (b) The content matter must come from the things in the environment.
- (c) Learners can easily explain, analyse the contents by activity.
- (d) The things which are required in different professions should be included in the curriculum.
- (e) Organisation of curriculum should be based on direct experience of the children.
- (f) Concepts should be cleared by giving enough real examples not giving so much importance on definition.
- (g) The content should be organised in the curriculum from simple to complex.

5.4 List of geometry contents at primary level

- (a) Solids such as - book, cube, parallelopiped, sphere etc.
- (b) Surface and their types.
- (c) Point, line - straight line and curved line.
- (d) Plane figure, geometric figure and their nature as - triangle, quadrilateral, circle.
- (e) Angle and their types.

5.5 Presentation - Solids

Materials required - Box, Book, Dice, Pipe, Can, Marble etc.

The teacher first distributes the materials among students and ask them name and number of surfaces of the

materials. Then make a table like the following -

Dice	–	cube	–	6 plane surfaces
Box	–	Parallelopiped	–	6 plane surfaces
Book	–	Parallelopiped	–	6 plane surfaces
Rod	–	cylinder	–	2 plane surfaces 1 curved surface
Ball	–	sphere	–	1 curved surface
Banana cone	–	cone	–	1 curved & one plane surface

So, we can say that things which have length, breadth, height and occupy some spaces are called solids. The upper part of these solids are called surface. A surface has length and breadth, but no height. There are different types of surfaces - Plane surface, Rough surface and Curved surface.

Plane surface - Surface which is regular but not rough is called plane surface.

Rough surface - Surface which is not regular is rough surface.

Example - Broken stone.

Curved surface - Surface which is smooth but curved is called curved surface. Example- Football, Marble.

Now teacher can ask following questions to classify the surfaces –

- (1) What type of surface is of mirror ?
- (2) How is the upper surface of a brick ?
- (3) What type of surface is of a marble ?
- (4) How many surface are there and what are they ?
- (5) What is the difference between plane surface and rough surface ?
- (6) What is the difference between rough surface and curved surface ?
- (7) Give one example of each of plane surface and curved surface.

5.6 Line

Draw two figures and then ask students about the figures. –



Straight line



Curved line

A line is a geometric figure which has length, but no breadth and height. There are two types of line - straight line and curved line. A line which does not change its direction is called straight line and the line which always changes its direction is called curved line.

The continuous motion of a point in a plane generates a line. Line is therefore, assembly of infinite number of points.

A plane is generated as a result of translation of a straight line.

Properties of straight line :

- (1) A straight line is constructed with infinite number of points.
- (2) It has length, but no breadth and height.
- (3) It does not change its direction.
- (4) It can be extended both sides endlessly.
- (5) A straight line can be drawn only on a plane surface.
- (6) A straight line intersects another straight line only at one point.
- (7) Only one straight line is drawn through two points.

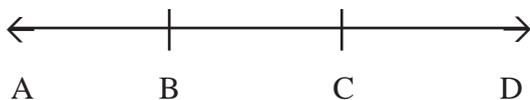
Properties of curved line :

- (1) A curved line is constructed with infinite number of points.
- (2) It has length, but no breadth and height.
- (3) It always changes its direction.
- (4) It can not always extend on both sides.
- (5) It can be drawn on plane surface, curved surface and rough surface.
- (6) One curved line can pass through a fixed point once or a number of times.
- (7) Two curved lines can intersect each other one or more times.

Above properties can be verified in the school at different functions of the schools such as – Annual sports, Drawing, Folding of paper etc.

When a straight line extends in one direction only starting from a point, the line is called a **ray** (in analogy with optical ray)

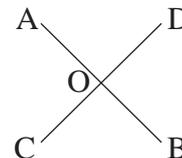
Line segment : Part of a line is called line segment.



In the above figure, \overleftrightarrow{AB} is the line and \overline{CD} is line segment.

Point : Where two straight lines intersect each other is called a point.

Two straight lines have intersected each other at O called point. Such points exist at the corner of a room, table, board etc. A true point has location but not length, breadth and height.



5.7 Angle and its classifications

The teacher can give the concept of angle in the following way of activity –

Materials required – Stick, pin.

Description –

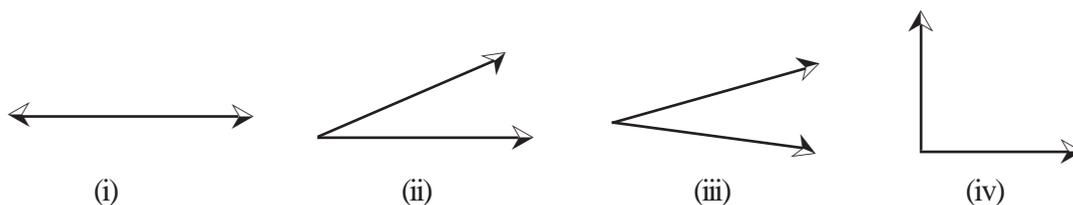
Step - 1 – Take two sticks of equal size.

Step - 2 – Fixed one end of the sticks with a pin.

Step - 3 – Hold one stick on the paper.

Step - 4 – Move free end of another stick.

Now you can see different types of figures are constructed like –



Now ask questions to your students –

- (1) In how many points two sticks meet ?
- (2) Are the gaps between two lines always same ?

So, if two straight lines moves in two directions from a fixed point, then the geometric figure is called angle. The fixed point is known as vertex and the line segments are called sides or arms of the angle.

Remember that degree ($^{\circ}$) is used for unit of angle.

From the above figures it is observed that the gaps between the sides are different. So according to the measures of the gaps there are five different types of angles. Such as –

- (1) Acute angle - whose measure is greater than 0° , but less than 90° .
- (2) Right angle - whose measure is 90° .
- (3) Obtuse angle - whose measure is greater than 90° but less than 180° .
- (4) Straight angle - whose measure is 180° .
- (5) Reflex angle - whose measure is greater than 180° but less than 360° .

5.8 Activities for classification of angles

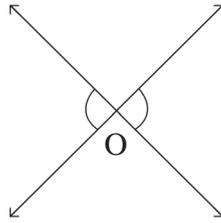
- (1) By folding or cutting of paper.
- (2) By arranging students in two rows during play.
- (3) By using teaching aids like - ruler, plane figure, solids etc.
- (4) Fencing, plotting of land.

5.9 Classification of angles according to special mutual relations of two angles

Complementary angles - If the sum of two angles is 90° , then each angle is complementary angle to other.

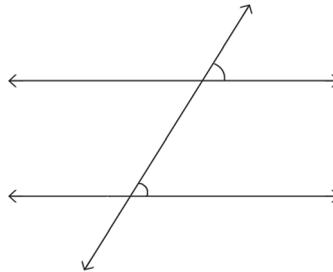
Supplementary angles - If the sum of two angles is 180° , then each angle is supplementary to others.

Vertically opposite angles - If two lines intersect each other, then the angles on opposite sides of the point of intersection is called vertically opposite angles.

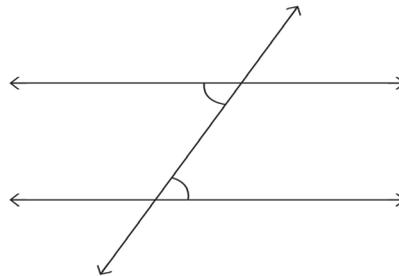


Corresponding angles -

If a straight line intersects two straight lines, then the angles on the same side of the transversal- one exterior and other interior are called corresponding angles.

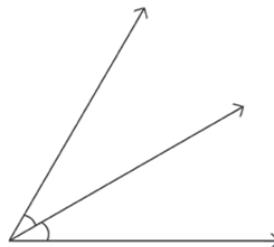


Alternate angles -



If a straight line intersects two straight lines, then the angles on opposite sides of the transversal - both being interior angles are called alternate angles.

Adjacent angles -



If two angles have a common vertex and a common arm and lies on both sides of the common arm, then the angles are called adjacent angles.

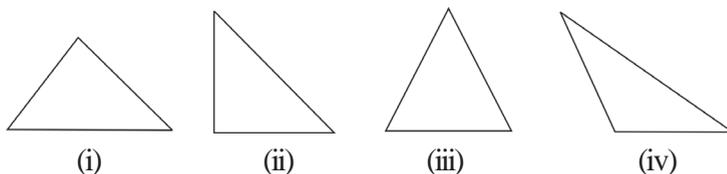
5.10 Plane Figure

A figure on a plane bounded by straight lines or curved lines is known as plane figure. As for example - Triangle, Quadrilateral, Circle, Polygon etc.

5.11 Triangle and its classification

Materials required - Ruler, Stick, String.

Description - Take sticks of different sizes. Tied the sticks with strings in the following way -



Questions -

- (1) What is the name of the plane figure bounded by three lines ?
- (2) Measure the sides of each figure and write their measures.
- (3) In which figure the measures of three sides are equal ?
- (4) In which figure one angle is obtuse angle ?

Equilateral triangle - A triangle whose three sides are equal.

Isosceles triangle - A triangle whose two sides are equal.

Scalene triangle - A triangle whose three sides are different.

Right angled triangle - whose one angle is 90° .

Acute angled triangle - whose three angles are acute angle.

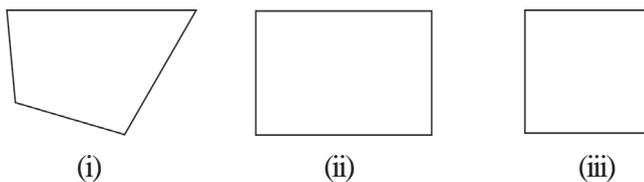
Obtuse angled triangle - whose one angle is obtuse angle.

5.12 Practical work on triangle

- (1) Construct triangle by cutting paper or hardboard.
 - (2) By folding paper.
 - (3) By constructing solid models with clay and showing their surfaces.
 - (4) By drawing triangles.
 - (5) By arranging students in triangular rows during play time.
-

5.13 Quadrilaterals and their classification

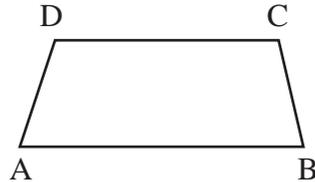
A plane figure bounded by four straight lines is called quadrilateral.



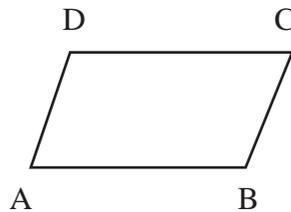
The above figures are examples of quadrilaterals.

A quadrilateral whose four sides are equal and all angles are right angles is called square fig. (iii). There are different types of quadrilaterals according to relation between sides and angles. Example - Square, Rectangle, Parallelogram, Rhombus, Trapezium.

Trapezium - A quadrilateral whose one pair of opposite sides are parallel is called trapezium. ABCD is a trapezium.



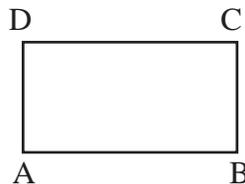
Parallelogram - A quadrilateral whose both pairs of opposite sides are parallel is called parallelogram. In the figure ABCD is a parallelogram, where $AB \parallel DC$ and $AD \parallel BC$.



Remember -

- (i) Opposite sides and opposite angles of a parallelogram are equal.
- (ii) The diagonals of a parallelogram bisect each other.

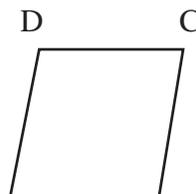
Rectangle - A parallelogram whose one angle is 90° is called rectangle. ABCD is a rectangle, where $\angle DAB = 90^\circ$.



Remember :

All rectangles are parallelogram, but all parallelogram are not rectangle.

Rhombus - A parallelogram whose adjacent sides are equal is called rhombus. ABCD is a rhombus whose $AB = AD$.



Remember -

- (i) Rhombus is a equilateral quadrilateral as its four sides are equal.
- (ii) If one angle of a rhombus is 90° , then it is square.

Practical Activity :

- (1) By folding paper.
- (2) By constructing solid models with clay and by showing their surfaces.
- (3) In the play ground.
- (4) By gardening, making road.

5.14 Circle and its parts

The closed path made by a moving point keeping equal distance from a fixed point on a plane surface is called circle.

Since circle is a closed curved line, it always makes a bounded plane area. This plane area is called circular disc. The fixed point inside the circle is called centre of the circle. The closed curved line is called circumference of the circle. The line segment joining the centre and any point on the circumference is known as radius of the circle. All radii of a particular circle are equal. The line segment joining any two points on the circle is known as chord of the circle. The chord passing through the centre of the circle is known as diameter. Diameter is the largest chord of the circle. $\text{Diameter} = 2 \times \text{radius}$. Any part of a circle is called arc of a circle. The diameter divides the circle in two equal parts. Each part is called semicircle.

Remember - Circumference is the greatest arc of a circle.

Teacher will draw circle with the help of a disc and explain the matter.

Practical work :

- (1) By drawing artistic paint (Alpana)
- (2) By drawing different pictures with circles.
- (3) By cutting paper for different paper work.
- (4) In sports of school.
- (5) By arranging students in different functions of the school, like, to hoist the flag etc.
- (6) By planting tree.
- (7) By constructing cone with clay and showing the circular surface.

5.15 Expected learning outcome

5.15.1 Line :

Knowledge :

- (a) Recollect the definition of line.
- (b) Recollect the types of line.
- (c) Remember the characteristics of lines.

Understanding :

- (a) Can draw different types of lines.
- (b) Can draw figures with lines.

5.15.2 Angles :

Knowledge :

- (a) Can recollect different types of angles.
- (b) Can recollect the unit of angle.
- (c) Can define different types of angles.

Understanding :

- (a) Can compare different types of angles.
- (b) Can identify different types of angles.

Application :

Can mention the angles and their characteristics in reality.

5.15.3 Triangle :

Knowledge :

- (a) Can recollect different types of angles and their definition.
- (b) Can name different types of angles.

Understanding :

- (a) Can mention similarities and dissimilarities among triangles.
- (b) Can identify different types of triangles.

Application :

- (a) Can draw different types of triangles.
- (b) Can use triangle to draw any figure.

5.15.4 Quadrilateral :

Knowledge :

- (a) Can recollect quadrilateral and their types.
- (b) Can define different types of quadrilateral.

Understanding :

- (a) Can compare different types of quadrilateral.
- (b) Can identify quadrilaterals.

Application :

- (a) Can mention the uses of quadrilateral in practical life.
- (b) Can explain different types of quadrilaterals.

Skill :

Can draw (construct) different types of quadrilaterals.

5.15.5 Circle :**Knowledge :**

- (a) Can define circle.
- (b) Can define different components of circle.

Understanding :

- (a) Can identify different parts of circle.
- (b) Can explain the relations between different parts of circle.

Skill :

- (a) Can draw circle.
- (b) Can use circle in daily life.

5.16 Check your progress

- (1) What do you mean by line and line segment ?
- (2) Name different types of angles.
- (3) Give definition of adjacent angles and vertically opposite angles.
- (4) What are the different types of triangles ?
- (5) Define quadrilateral and classify them.
- (6) What is the greatest chord of a circle ?
- (7) What is equilateral triangle ?
- (8) Name the quadrilateral whose diagonals are equal and bisect each other.

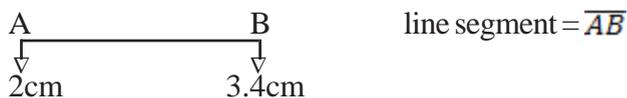
5.17 Line segment

A line is generated as a result of continuous motion of a point on a plane. So a line is a conglomeration of infinite number of points. The line which extends on two sides without changing the direction is called straight line. The line which has no definite direction is called a curved line. A plane is generated as a result of translation of a line.

Considering a two points on a ruler they should be connected by drawing a line along the ruler.

Measurement of length of a line segment.

A student sets the ruler along the segment of the line and adjusts a point of the ruler with the initial point of the segment. He will then observe the other point of the ruler which meets the terminal point of the segment. The difference of reading between two points is the length of the line segment.

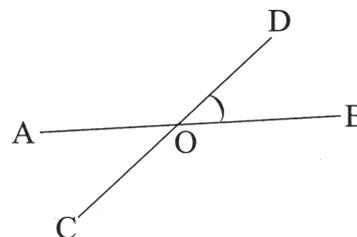


Then the length of the segment = $3.4\text{cm} - 2\text{cm} = 1.4\text{cm}$

5.18 Drawing of angles

The student will draw two so that they meet at a point

Here four angles have been generated which are AOC, BOC, BOD, AOD. We have to measure the angle BOD. For this a learner has to use the protractor of a geometry box. The lower line of the protractor has to be placed along the line OB and the midpoint of the line has to be set on O. At this stage the line of the protractor coinciding with OD has to be selected and the value noted on the protractor near the line has to be recorded. The measure of the angle has to be recorded in the following way. Angle $BOD = x^\circ$ where x is the measure of the angle (from protractor) and $(^\circ)$ is the unit. If a student likes to draw the angle of a particular value, s/he can approach in the following ways: By using



- (1) Scale compass of the geometry box
- (2) The protractor and ruler of a geometry box.

5.18.1 For primary students the following process is shown:

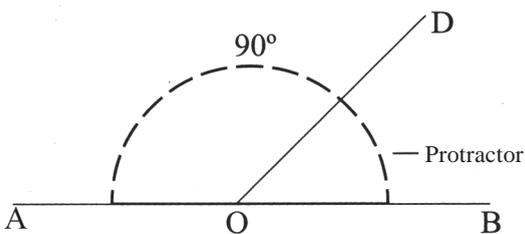
First you should draw a line AB, let a point O be taken on it.

On O the lower line of the protractor has to be set along line A

AB. Under this condition if acute angle has to be drawn, angle

between 0° to 90° has to be selected on protractor and a point

to be marked by P near the acute angle. When PO is joined the acute angle would be drawn.

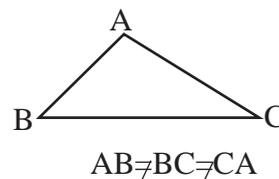
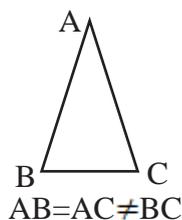
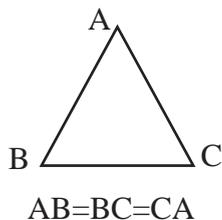


For drawing right angle a point P is to be placed near 90° of the protractor. When P and O is joined we get a right angle. For drawing right angle set square may also be used. By folding paper right angle may be drawn.

5.19 Drawing triangle:

For drawing triangle three intersecting straight line have to be drawn with the help of a ruler. The name of the points are A, B & C. The name of the triangle is therefore ABC, whose angular points are A, B & C. On the basis of sides three types of angles could be drawn: they are Equilateral triangle, Isosceles triangle & scalene triangles.

To draw Equilateral triangle: It is better to use compass to draw equilateral triangle. First the learner is to draw a line segment BC. With BC as a radius, the learner would draw arcs centering B & C. Let the two arcs intersect at A. On joining AB & AC an equilateral triangle ABC will be obtained. For drawing isosceles triangle two methods may be used: (1) By using simple scale or ruler, (2) by using compass.

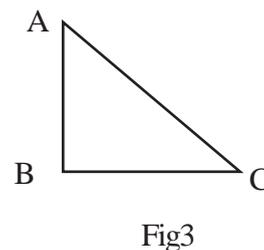
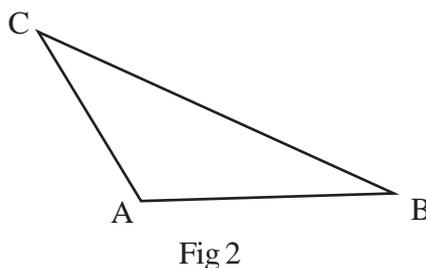
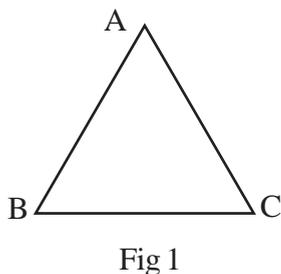


For scalene triangle students may use only ruler.

5.19.1 Triangles with different types of angles

- (1) Acute Angled Triangle
- (2) Right Angled Triangle
- (3) Obtuse Angled Triangle

Acute Angled Triangle : Drawing acute angled Triangle **Fig 1:** Angles of the triangles should so selected that each angle of the triangle is less than 90° but sum of the angle becomes 180° . A straight line segment AB is drawn. At its two ends two selected acute angles are drawn. The third angle so produced would be would also be an acute angle.



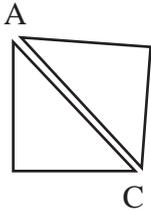
Obtuse Angled Triangle Fig 2: A line segment AB is drawn. At one end say A, an obtuse angle ($>90^\circ$ & $<180^\circ$) is drawn with the help of a protractor. The second line attached to the angle is AC. On joining BC we get an obtuse angled triangle. Here angle $CAB > 90^\circ$ but $< 180^\circ$

Fig 3: Right angled triangle may be drawn with the help of set square in the instrument box

In the above right angled triangle CAB, angle $ABC = 90^\circ$

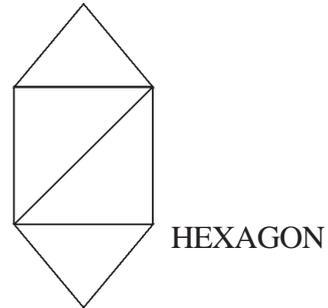
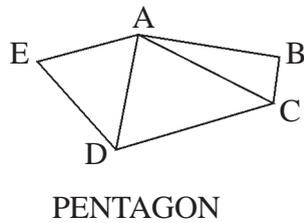
5.20 Drawing a polygon

For drawing polygon the students will start with quadrilaterals. The students may draw quadrilateral in a play way mood with the help of setsquare available in geometry box. For this the student may collect more than one set square from the geometry box of the friends.



Here two similar set squares of geometry box have been used to draw the quadrilateral ABCD.

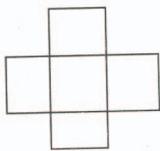
The above diagram may be drawn with the help of card board triangles instead of setsquare of geometry box. With the help of card board triangles a student can draw pentagon, hexagon etc.



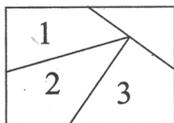
In the pentagon three triangles are required and in the hexagon 4(four) such triangles are required. In forming the designs, decoration of floors, walls, cover design of books and in fine arts different lines, geometrical diagrams etc have to be designed.

5.21 Drawing Diagrams with Basic Geometrical Figures

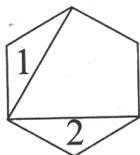
For drawing simple diagrams some simple geometric figures may be assembled. Examples are shown below:



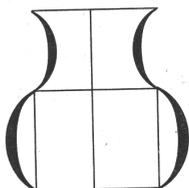
To draw square by the intersection of two rectangles



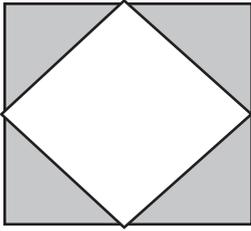
With the help of a three quadrilaterals and a triangle a square is drawn.



With the help of two triangles and a quadrilateral a hexagon is drawn.



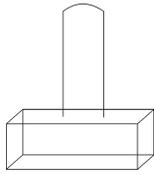
Four circular arcs and two squares form a pitcher



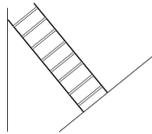
In a square another square is placed by turning it through 45° .
 In addition to this, four isosceles triangles are produced within it.

5.22 Constructing model with solid bodies

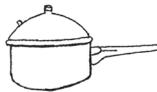
In primary mathematics different basic geometrical solid bodies are presented before the students. e.g. parallelo piped (bricks, match box), cube (dice), cylinder, prism, sphere or globe etc. Different models can be produced by combining these solid bodies. Some examples of these models are given below :



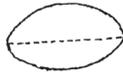
Monument: A solid cylinder has been erected over a rectangular parallelo piped



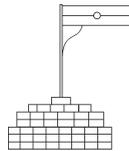
Ladder: A set of rectangles makes a ladder.



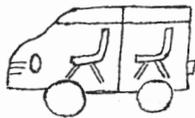
Pressure Cooker: A cylinder, an arc and a straight line makes a pressure cooker



Lens; Two spherical surfaces make a lens



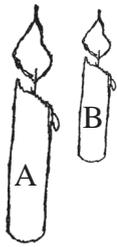
Flag Stand : It is made up of bricks and a pole supporting the flag. The pole is a cylinder, the bricks are parallelo piped and the flag is a rectangle



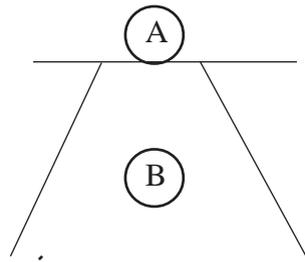
Coach of a Train : One curved surface, one parallelo piped, two circular discs (wheels) four triangles,

5.23 Figural and spatial relationship

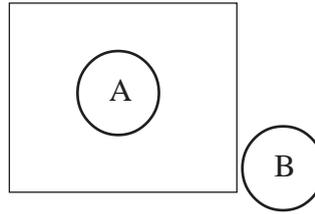
Alfred Binet while determining intelligence of the children gave importance on shape and spatial relationship. Now-a-days importance is given on spatial relationship while assessing Aptitude of the students. So such relationship should be discussed at elementary level. So such relationship should be discussed at elementary level. For determining the spatial relationship at least two objects are required. One of them is to be treated as reference object with respect to which position of other is determined. Some of the spatial relations are (1) small & large, (2) upper & lower, (3) inside-outside, (4) near distant, (5) forward & backward, (6) front and side (7) middle & side.



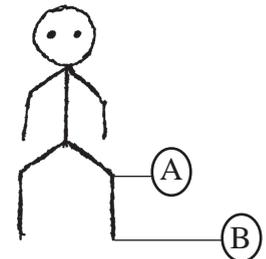
A Bigger, B smaller



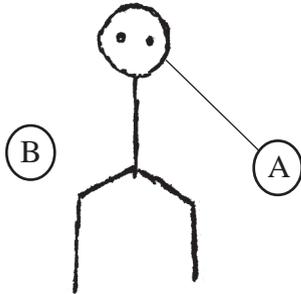
A above, B below



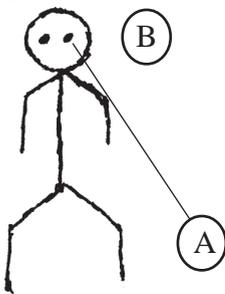
A- within, B-Outside



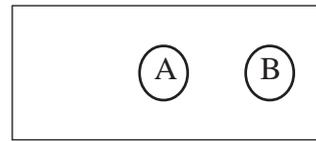
A near, B far



A in front , B at side



A in front, B at the back

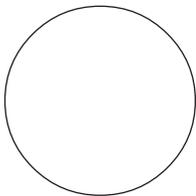


A at the middle, B at the side

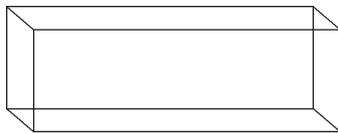
5.24 Shape

They are discussed with following figures:

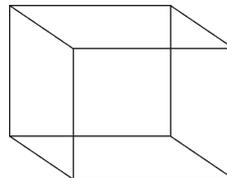
Since birth child gradually develops the concept of different colours and shapes. The shapes that capture the attention of the students are mainly three dimensional objects which comprise: Spherical Objects, Parallelopiped, cube, cylinder, cone ,circular objects etc. Then he switches over to 2D objects-like triangles, quadrilaterals, polygons, and irregular surfaces and bodies like pieces of papers, broken tiles. These are followed by straight and curved lines.



Spherical Objects



Parallelo piped



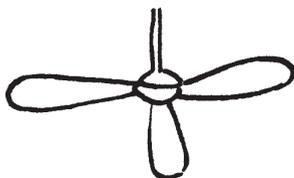
cube



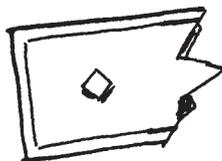
Cylinder



Cone



Circular objects



Irregular surfaces



Curve line



Straight line

5.25 Different types of lines

Curved lines can be observed in hair, in dresses. Apart from these motion of birds, cats, dogs, hares are the examples of curved lines.

Straight lines can be observed in freely falling bodies under gravity. Teacher may bring illustrated charts and diagrams as TLM to demonstrate the lines.

For visual observation charts can be used. For visual observation and touch sensation concrete objects may be handled. Constructivist approach can be used for description of the figures

5.26 Unit Summary

This unit discussed

- Concept of point, line and plane
- Different types of line, line segment, ray, triangle, polygon
- Drawing line segment and their measurement; drawing of triangle, polygon and necessary measurement
- Drawing different sketches with the help of line, angle, triangle
- Preparing model with the help of solid geometrical bodies.
- Different shapes and spatial relation and their teaching learning method

Exercises:

- (1) Define: point, line, plane, triangle, angle
 - (2) What is the relation between line and a point?
 - (3) How would draw an acute angle?
 - (4) What do you mean by basic shape?
 - (5) Write the name of four solid geometrical structures
-

5.27 Know your progress

- Write the characteristics of a straight line.
- What is the relation between chord and diameter?
- How many triangles are required to draw a quadrilateral?
- How many triangles are required to draw pentagon?
- How can you draw a square with help of two same size rectangles?
- Drawing necessary diagrams demonstrate the different positions of an object : inside & outside, up & down, near & far.

Lesson Unit- 6

EVALUATION

Lesson Unit

- 6.0 Evaluation
- 6.1 Introduction
- 6.2 Objectives
- 6.3 Characteristics of Evaluation
- 6.4 Utility of Evaluation in Mathematics Education
- 6.5 Different Techniques of Evaluation in Mathematics
 - 6.5.1 Written Test
 - 6.5.2 Oral Test
 - 6.5.3 Practical Test
- 6.6 Sample of different Objective Type Questions
- 6.7 CCE in Mathematics
 - 6.7.1 Formative Evaluation
 - 6.7.2 Planning for Unit Test
 - 6.7.3 Unit Test Paper
- 6.8 Diagnostic Tests for Exploring Weakness in Learning
 - 6.8.1 Place of Diagnostic Test in Mathematics
 - 6.8.2 Techniques of Constructing Diagnostic Test in Mathematics
- 6.9 Summary
- 6.10 Check your Progress
- 6.11 Exercises

6.1 Preface

Evaluation is the continuous measurement of gradually increasing personality development of a learner. In other words evaluation is such a measurement operation by which, in short interval, through continuous test acquired ability of a learner can be evaluated and at the same time deficiencies and lapses can be removed. Thus total success in both teaching and learning can be arrived at through evaluation.

6.2 Objective

After reading this chapter we can know the following matters.

- (1) Features of evaluation.
 - (2) Necessity of evaluation in the teaching of mathematics.
 - (3) Different techniques of evaluation in mathematics.
 - (4) Continuous evaluation in primary education.
 - (5) Role of tests to determine the weakest points of the learners.
 - (6) Fear of mathematics and its removal.
 - (7) Reasons of backwardness in mathematics and finding out of its remedy.
 - (8) A test to diagnose the weakness in mathematics.
 - (9) Assessing needs of Advanced learners of mathematics.
-

6.3 Characteristics of Evaluation

On analyzing of different aspects of evaluation comprehensiveness of evaluation in the primary stages will be readily apparent.

- (1) Evaluation is a continuous process and by frequent tests in short interval weakness and defects of students in mathematics can be detected.
- (2) Evaluation is a process of determining development of a learner. Evaluation through number of tests control defects and lapses of a learner and also takes necessary steps for the removal of the same. By this way of evaluation total success of learner can be upgraded. To measure acquired abilities of a learner is not the main objective but the main objective is to rectify the defects and the lapses of the learner constantly.
- (3) Evaluation is a comprehensive process. It does not only evaluate the acquired skill in different lessons, but also measures and refines physical & mental characteristics of the learners.
- (4) As evaluation is a determining operation of quality of a learner, he can know all about his success and failure every moment and therefore he can himself rectify all defects and lapses without any mental stress.
- (5) In the evaluation process all tests are mostly ‘objective based’ and thus the main objectives of education can be arrived at easily.

6.4 Necessity of Evaluation in the Teaching/Learning of Mathematics

The teaching/learning of mathematics has a close relationship with the educational experience and evaluation of a learner. Experiences are arranged depending on the objectives. Evaluation system is to note what changes have been made in the above experiences of the child and what not. Learning of a child is not confined in the text book only. Experience acquired in the class, school, environment, home and social environment is the subject matter of learning of mathematics. In the learning of mathematics only cognitive knowledge is not gained in his practical life. Simultaneously value and attitude are formed. Besides, provision of remedial lessons is to be used in respect of those learners who have retrograded in their daily teaching.

This evaluation cannot be done by the existing evaluation system, because the existing examination can only evaluate the attainment of past knowledge, but it can not evaluate gradual changes of personality, attitude, thinking, interest etc. The objective of evaluation is to see whether learning of mathematics is being rightly implemented or the objective of education is made realistic. So, by means of applying different methods like written examination, oral test, practical works etc. the learners are to pass through continuous evaluations.

6.5 Different Techniques of Evaluation of Mathematics

In general techniques of evaluation at the primary stage are written examination, oral test and practical examination.

6.5.1 Written Examination

In this examination the answers of the questions are given on the answer scripts in writing. Generally 90% of the total marks are evaluated by written examination, which will obviously be knowledge-based, understanding-based, application-based and skill-based. If the questions are competency-based then the answer key is to be prepared for evaluation. The questions are in general of three types: -

- (a) Very short answer based
- (b) Short answer based
- (c) Essay type

Very short answer based questions of 1 mark, each answer will be given in one sentences only. The value of short answer based questions will be of 2-3 marks. Questions will be knowledge-based, understanding-based and skill-based. Essay type question will be of 3 - 5 marks. Through these questions mainly a learner's knowledge-based, understanding based and application-based attainments are tested.

6.5.2 Oral Test

In the type of oral test the learners are to give immediate answer to the questions of the teachers in their presence. 10% marks are allotted for this oral examination. Special vigil to be given so that the questions are understanding-based and application-based. In class I and II frequent oral examinations should be arranged. Aspects of mathematical evaluation are - a) concepts of mathematics, b) reasoning and problem solving ability c) fluency and correctness. To test their abilities oral questions are very much necessary. For testing the conception of the learners they have learnt, their interest, attitude and opinions, oral questions are necessary. The teacher will prepare the oral questions by thinking well in advance. Value of each question and probable answer should be done properly. Oral questions will be very short-answer type.

A few questions may be short-answer based. For the solution of a definite mathematical problem which methods and techniques have been followed and why can be known through oral questions. Learner's ready wit and his technique of using mathematical processes in practice can be ascertained through oral questions.

Importance of oral test–

- (1) Good relationship between teacher and learner develops.
- (2) Spontaneity of speech develops.
- (3) Oral test is more effective in the class I and II for the learners not able to give answers in writing.
- (4) Immediate correction is made if the answer is incorrect.
- (5) Immediate power of thinking can be developed.
- (6) In short time evaluation of many learners can be done.

6.5.3 Practical Examination

There is no provision of practical examination of mathematics meant for result sheet. Yet, to develop the mathematical habits and practices and to make the objective of the mathematics very often practical works of learner in mathematics are evaluated as mentioned below.

- (1) In the case of productive and creative works mathematical accounts are applied. By this method abstract methods of mathematics are practised and geometrical conception becomes wider.
- (2) If the mathematical practices be done through works and games, a learner's inherent interest in mathematics increases and concrete conceptions is formed. In this case qualitative evaluation can be done.
- (3) Different types of mathematical TLMs can be made. This also comes in the process of evaluation.
- (4) In many schools mathematical calculations and geometrical diagrams are done by the learners. In such cases also evaluation should be done properly.

Through practical examinations the following aspects are evaluated:–

- (a) Interest of the children develops due to works in mathematics.
- (b) Skill of using mathematics.
- (c) Skill of making TLMs.
- (d) Application power of mathematical conception in practical works.

6.6 Samples of Different Objective Type Questions

- (1) Recall type.
 - a. How many parts are there in a simple fraction?
 - b. If denominator is 5 and numerator is 4 what will be the fraction – proper or improper?
 - c. Is the value of improper fraction greater than one or less than one?
- (2) Completion type.
 - a. In case of proper fraction denominator is.....than numerator.(greater/less).
 - b. $\frac{1}{2} = ? / 80$
 - c. A fraction multiplied by its..... gives result 1.

3) MCQ.

Select the right answer.

(i) (a) $\frac{2}{3} > \frac{3}{4}$ b) $\frac{2}{3} < \frac{3}{4}$ c) $\frac{2}{3} = \frac{3}{4}$ d) None

(ii) Which of the three following diagrams is equal to $\frac{2}{3}$ parts.

(a)  b)  c)  d) None

4) True or False type.

a. Is the statement true or false?



Marked part is equal to fraction $\frac{1}{5}$

b. If the same number is added to the numerator and denominator of a proper fraction, will the new fraction be greater than or less than the first fraction?

5) Classification Type.

a. Select from the following which does not belong to the same category as of others.

(i) $\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{5}$ $\frac{6}{5}$

(ii) $\frac{3}{2}$, $\frac{4}{3}$, $\frac{5}{4}$, $\frac{5}{6}$

6) Matching type.

Match the following with left hand side and right hand side by giving example.

LHS	R.H.S. (for example)
Proper fraction	$1\frac{2}{3}$
Mixed fraction	$\frac{1}{10}$
Improper fraction	$\frac{4}{3}$

6.7 CCE

In the existing curriculum one of the important factors is 'No detention policy'. As this is under RTE act this policy is followed all over India. To make the no detention policy successful continued evaluation policy is introduced and also by continued evaluation errors and lapses of the learners are identified and then by swift and quick correction method learners' all round development is to be done. There are two aspects of continued evaluation :-

- (1) Achievement based evaluation.
- (2) Formative evaluation.

The achievement based evaluation gives the measurement of achievement or acquired ability of the students. Formative evaluation is mainly a process of detecting defects, weakness and lapses. These lapses can be removed by giving revision lessons to help acquiring all-round success of the learners.

6.7.1 Formative Evaluation is done in two ways

- At the end of daily class lesson and on the basis of measurement of acquired ability through written and oral test, next lesson plan is to be prepared.
- Unit evaluation means evaluation of a chapter or a part of big chapter. At the end of a mathematical operation, a test of marks 10 is taken. On analysis of the result written and oral tests will be arranged. In preparing a unit evaluation sheet if the evaluation is done properly all learners can get in success in varied extent. And thereafter, the teacher will proceed to the next steps. The cause of backwardness of the learners in mathematics can be removed through such evaluation techniques.

6.7.2 Planning of unit evaluation

Class-III

Lesson Unit – Factors and Multiple.

Marks-20

Subject - Mathematics

Time - 30 minutes.

Division of marks according to sub units		Number of divisions on the basis of learning abilities				Division on the basis of type of questions			
Sub units	Marks	Knowledge Based	understanding Based.	Application Based.	Skill Based.	objective	Very short	Short	Essay type.
a) Concept of Factors	5	2	3	0	0	2	3	-	-
b) Concept of Multiple	6	2	2	2	0	1	3	2	-
c) Relation between a) and b)	3	0	3	0	0	-	1	2	-
d) Practical problem	6	0	0	6	0	-	-	2	4
Total	20	4	8	8	0	3	7	6	4
Percentage	100%	20%	40%	40%	0	15%	35%	30%	20%

6.7.3 Unit Evaluation Sheet

Class-III

Lesson Unit-Multiplicand and Multiple.

Marks-20

Subject - Mathematics

Time - 30 minutes.

1. Select the correct answer.
 - (i) Prime Factors of prime number is
 - a. Innumerable
 - b. One
 - c. Two
 - (ii) Prime Multiple of a composite number is
 - a. One
 - b. Innumerable
 - c. Two
 - (iii) Number of Multiplicand of a composite number is
 - a. More than one
 - b. More than two
 - c. Two
 - (iv) Which two numbers of the following number pairs have numbers prime to each other?
 - a. 4, 5
 - b. 2,6
 - c. 7, 14
 - (v) Factor of all numbers is
 - a. One
 - b. Zero
 - c. None
2. Fill in the blanks:
 - (a) Multiplicand and Multiple of what number are mutually
 - (b) A given number divisible by any other number is also divisible by other number's.....
 - (c) Factors of a prime number are only in number.
3. Give answer in one word:-
 - (a) What is Multiple?
 - (b) What is the difference between the Factor and Multiple?

- (c) You want to divide some mangoes equally among 4 children. What is the highest number of mangoes but less than 50 will be necessary?
4. If some lemons are equally divided among 15 persons 7 remains in excess. If this lemons are divided equally among 5 persons, how many lemons in excess? At least how many lemons are necessary?

6.8 Place of Test to determine the Weakest Positions of Learners

In a formative evaluation measurement of achievement of a learner is secondary. Primary objective is to determine his/her the errors or weakness. The main objective of this diagnostic test is to identify the area of weakness and errors of a learner and to find out the actual cause of it. With a view to bringing all round success of a learner in acquiring educational outcome of this sort diagnostic test is necessary, and very much important. For this reason unit test is necessary.

As content of a unit test is less and time of administration is short, this sort of evaluation is the best for immediate use. If this unit test in formative evaluation goes on continuously, the child will never slipback in learning difficult concepts.

Formation of a unit test is tri-dimensional : - a) division of number according to type of questions. In such a case number of very short answer-type questions and short answer-based questions are more in number and so different types of questions are asked. As a result it becomes easier and proper to find out the errors and lapses of a learner, b) questions based on learning ability i.e. of knowledge, understanding, application and skill are set for which all types of learning can be tested. As in a diagnostic test there are priorities of setting understanding-based and application-based questions, many other important aspects beyond the text books can be judged and analysed which is a very important aspect of education. Not only to determine the weakest spot but also to find out the cause of weakness of a learner is the objective of preparing unit test. (c) According to contents (or sub-units) the scores are distributed.

6.8.1 Diagnostic test of determining Weakness in areas of Mathematics

In the matter of diagnostic test of a learner, weak points and their causes are determined. And for these an enquiry is done through diagnostic test. And by this diagnostic test the areas of weakness of a learner can be detected and identified and attempt is made to remove by applying different techniques of teaching. In case of achievement test and diagnostic test they have in many occasions structural similarity but different in their objectives. The objectives of achievement test is to measure progress of a child and the main objective of a diagnostic test is to identify the weakest points and to find out the causes of such weakness. One of the most important characteristics of the curriculum in the present conventional primary education is the continuous evaluation methods through which all learners are taken towards total success. To maintain this continuity diagnostic test has much importance. The processes of mathematics are correlated. If there are any weak points in any process, then in other methods also the learners will be weak and will develop phobia for mathematics and the student will lag behind. In order to make a diagnostic test, the teacher shall be well equipped with the characteristics of this diagnostic test so that the purpose of this diagnostic test is fulfilled.

6.8.2 Characteristic of Diagnostic Test

- (a) Over a small area of a subject diagnostic test is to be prepared.
- (b) In a diagnostic test number of questions shall be much more.

- (c) Questions of the diagnostic test will be objective and very short answer type.
- (d) Questions of diagnostic test should be based on the probable mistakes done by the learners.
- (e) Questions of the diagnostic test should include all type of learning abilities.
- (f) On the basis of a particular mathematical process and conception diagnostic test is to be prepared.
- (g) Questions of the diagnostic test will be much more analytic and practical.
- (h) In the administration of diagnostic test adequate time is to be allotted.
- (i) Defects and weaknesses of a learner can be detected through this diagnostic test.

The teacher will prepare a sample of a diagnostic test following the above instruction.

6.9 Summary

- What is evaluation? What are their characteristics?
 - What is the necessity of the evaluation?
 - What are the different techniques of evaluation of mathematics?
 - What is the necessity of practical test in mathematics?
 - Conception of continuous evaluation process.
 - Role of diagnostic test in determining weakest areas of the learner.
-

6.10 Evaluate Your Progress

- (a) Write down the characteristics of evaluation.
 - (b) What is the necessity of evaluation in the teaching of mathematics?
 - (c) What are the techniques of evaluation in mathematics?
 - (d) What is the importance of oral test?
 - (e) Write down what you mean by continuous evaluation.
 - (f) What types of questions can be set in the test?
-

6.11 Exercises

- (a) What are 'achievement and evaluation'? What is the difference between them? Explain.
- (b) Write down the characteristics of evaluation.
- (c) Discuss the importance of different techniques of evaluation of mathematics.
- (d) What should be the nature of questions for the evaluation? Give example.
- (e) In the present educational system what type of evaluation is more acceptable?
- (f) How would you use the tests to identify the weakest points of the students in mathematics?